

Comment on “Possible resolution of the Casimir force finite temperature correction “controversies” ”

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Abstract

The recently suggested modification of the transverse electric contribution to the Lifshitz formula (S. K. Lamoreaux, arXiv:0801.1283) is discussed. We show that this modification is inconsistent with the data of two precise experiments, and violates the Nernst heat theorem. The preprint’s suggestion concerning the resolution of the “apparent violation of the Third Law of Thermodynamics” is shown to be incorrect.

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The preprint [1] suggests a modification for the contribution of the transverse electric modes to the Lifshitz free energy in the configuration of two conducting semispaces at a temperature T separated by a gap of thickness d

$$\mathcal{F}^{\text{TE}} = \frac{k_B T}{2\pi} \sum_{n=0}^{\infty}{}' \int_0^{\infty} q dq \ln \left[1 - \left(\frac{\gamma_{1n} - \gamma_{0n}}{\gamma_{1n} + \gamma_{0n}} \right)^2 e^{-2\gamma_{0n}d} \right]. \quad (1)$$

Here, $\gamma_{0n}^2 = q^2 + \xi_n^2/c^2$, ξ_n are the Matsubara frequencies, and the standard expression $\gamma_{1n}^2 = q^2 + \varepsilon(i\xi_n)\xi_n^2/c^2$ [2] is replaced with

$$\gamma_{1n}^2 = q^2 + \lambda^{-2} + \varepsilon(i\xi_n)\frac{\xi_n^2}{c^2}, \quad \lambda^{-2} = \frac{e^2 c_t}{\bar{\varepsilon} \epsilon_0 k_B T}, \quad (2)$$

where λ is the Debye-Hückel screening length [$\varepsilon(\omega)$ is the permittivity of a conductor, c_t is the total carrier concentration, ϵ_0 is the permittivity of vacuum, and $\bar{\varepsilon}$ is the dielectric constant due to core electrons]. According to Ref. [1], the effect of Debye-Hückel screening in accordance with Eq. (2) leads to the same zero-frequency contribution in Eq. (1) as for ideal metals and resolves the contradiction between the Lifshitz formula combined with the Drude model and the experimental data of Ref. [3]. (Note that there was a discussion [4, 5] about the comparison between the experimental data of Ref. [3] and theory.) Below we demonstrate, however, that the suggested modification is not only inconsistent with two other more precise experiments on the measurement of the Casimir force but also violates the Nernst heat theorem.

Using the approach of Ref. [1], we have computed the Casimir pressure between two Au plates and the Casimir force between an Au sphere and a Si plate in the experimental configurations of Refs. [6, 7]. The values $c_t \approx 5.9 \times 10^{22} \text{ cm}^{-3}$ and $c_t \approx 3.2 \times 10^{20} \text{ cm}^{-3}$ at $T = 300 \text{ K}$ were used in the computations for Au [6] and highly doped n -type Si [7], respectively. We have included the term λ^{-2} , as in Eq. (2), only for $n = 0$ (as noted in [1], λ is frequency-independent only at low frequencies $< 10^{10} \text{ Hz}$). The inclusion of any nonzero λ^{-2} in the Matsubara terms with $n \geq 1$ would only increase the magnitudes of the theoretical pressure and force, and thus would increase the disagreement between experiment and theory. The standard contribution of the transverse magnetic modes was employed [2]. In Fig. 1(a) the differences between the computed theoretical Casimir pressures and the experimental data of Ref. [6] are shown as dots at different separations. In Fig. 1(b) the differences between the computed theoretical Casimir forces and respective data of Ref. [7] are presented. Solid lines in both figures indicate the boundaries of the 95% confidence

intervals. Dashed lines in Fig. 1(b) show the boundaries of the 70% confidence intervals. As it is seen in Fig. 1, the experiment [6] excludes the theoretical approach of Ref. [1] within the separation region from 170 to 450 nm at a 95% confidence level. The experiment [7] excludes this approach with a 95% confidence at separations from 62 to 82 nm and with a 70% confidence within a wider separation region from 62 to 100 nm.

Reference [1] claims to solve the thermodynamic inconsistency in the theory of the thermal Casimir force. As proved in Ref. [8], and independently confirmed in [9], the Lifshitz formula combined with the Drude model violates the Nernst heat theorem in the case of perfect crystal lattices. The statement of Ref. [1] to the contrary is based on errors. Reference [1] does not take into account the fact that T appears in the Casimir force calculation not only through the factor $\exp(\hbar\omega/k_B T)$ but also through the temperature-dependent dielectric permittivity $\varepsilon(\omega, T)$. We emphasize that this is actually the case for the Drude model. The behavior of the free energy when T goes to zero can be investigated using the Abel-Plana formula

$$\sum_{n=0}^{\infty} f(n) = \int_0^{\infty} f(t) dt + i \int_0^{\infty} \frac{f(it) - f(-it)}{e^{2\pi t} - 1} dt. \quad (3)$$

The second term on the right-hand side of this equation is not taken into account in [1]. Although it goes to zero when T vanishes, its derivative with respect to T may not vanish when T goes to zero. It is precisely this term which determines the nonzero value of the Casimir entropy at $T = 0$ K in the case of the Drude model [8]. By repeating the derivation of Ref. [8] we conclude that the approach of Ref. [1] also violates the Nernst theorem, as it leads to the following value of the entropy at $T = 0$ K:

$$S(0) = \frac{k_B}{4\pi} \int_0^{\infty} q dq \ln \frac{1 - \left(\frac{q - \delta_1}{q + \delta_1}\right)^2 e^{-2qd}}{1 - \left(\frac{q - \delta_2}{q + \delta_2}\right)^2 e^{-2qd}} < 0. \quad (4)$$

Here, $\delta_1 = (q^2 + \tilde{\lambda}^{-2} + \omega_p^2/c^2)^{1/2}$, $\delta_2 = (q^2 + \tilde{\lambda}^{-2})^{1/2}$, ω_p is the plasma frequency, $\tilde{\lambda} = [\epsilon_0 E_F / (3e^2 c_t)]^{1/2}$ is the screening length at low temperature in the Thomas-Fermi approximation [10], and E_F is the Fermi energy. Note that in the derivation of Eq. (4) all Matsubara terms with both polarizations were taken into account so that the TE modes with $n \geq 0$ have been modified, as prescribed by Eq. (2) suggested in Ref. [1].

To conclude, we have compared the approach of Ref. [1] with the experimental data of two recent precision experiments and found it to be inconsistent with these data at a high

confidence level. This approach also violates the Nernst heat theorem for a perfect crystal lattice, and is thus inconsistent with thermodynamics.

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- [1] S. K. Lamoreaux, arXiv:0801.1283.
 - [2] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics*, Part 2 (Pergamon Press, Oxford, 1980).
 - [3] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).
 - [4] A. Lambrecht and S. Reynaud, Phys. Rev. Lett. **84**, 5672 (2000).
 - [5] S. K. Lamoreaux, Phys. Rev. Lett. **84**, 5673 (2000).
 - [6] R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, Phys. Rev. D **75**, 077101 (2007); Eur. Phys. J. C **51**, 963 (2007).
 - [7] F. Chen, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, Phys. Rev. Lett. **97**, 170402 (2006).
 - [8] V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, and C. Romero, Phys. Rev. A **69**, 022119 (2004).
 - [9] F. Intravaia and C. Henkel, arXiv:0710.4915; J. Phys. A: Math. Theor, 2008, to appear.
 - [10] J.-N. Chazalviel, *Coulomb screening of mobile charges: applications to material science, chemistry and biology* (Birkhauser, Boston, 1999).

Figures

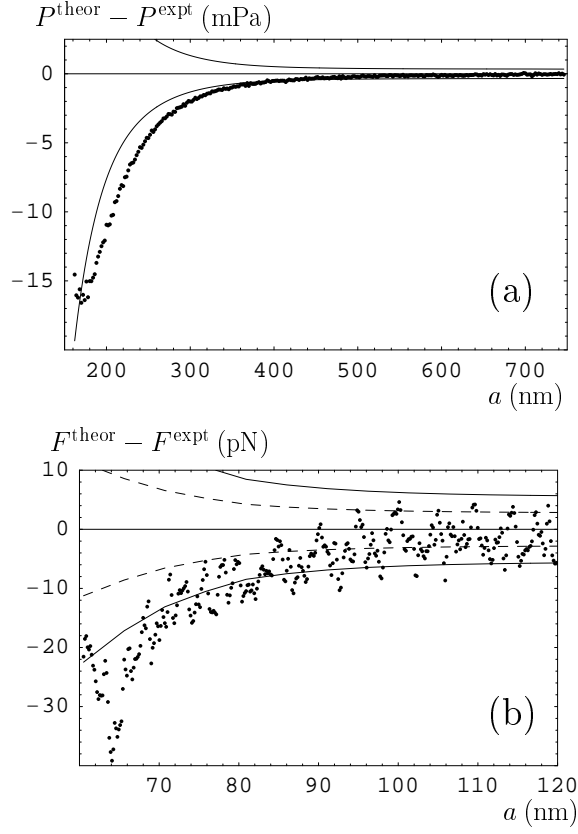


FIG. 1: Differences between theoretical (using the approach of Ref. [1]) and experimental Casimir (a) pressures and (b) forces versus separation. The experimental data are taken from (a) Ref. [6] and (b) Ref. [7]. Solid and dashed lines indicate the borders of 95% and 70% confidence intervals, respectively.